

1

Given a theory with a single real scalar field (and any interaction). Show that the 2-point function obtained from the effective action:

$$\Gamma^{(2)}(x, y) = \frac{\delta^2}{\delta\phi_\alpha(x) \delta\phi_\alpha(y)} \Gamma[\phi_\alpha]$$

effective action

$$\Gamma[\phi_\alpha] = W[J] + J \cdot \phi_\alpha$$

is the inverse of the full propagator:

$$\int d^4z \Gamma^{(2)}(x, z) G_2^c(z, y) = \delta(x - y)$$

(this was covered in QFT I, so this is mostly a forced revision)

(the definitions above are valid in Euclidean space, factors of +/- i appear in Minkowski, but feel free to do everything in Euclidean)

2

Consider the following Lagrangian:

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m_{\phi_0}^2 \phi^2 + \bar{\Psi} (\not{\partial} - m_{e_0}) \Psi - i g_0 \bar{\Psi} \gamma^5 \Psi \phi$$

re-write the Lagrangian doing the mass and field renormalization (obtaining something analogous to the first Lagrangian in page 22) and then write the Feynman Rules in terms of these new variables (note that you don't need to redefine  $g_0$  at all, we are leaving that for next lecture)